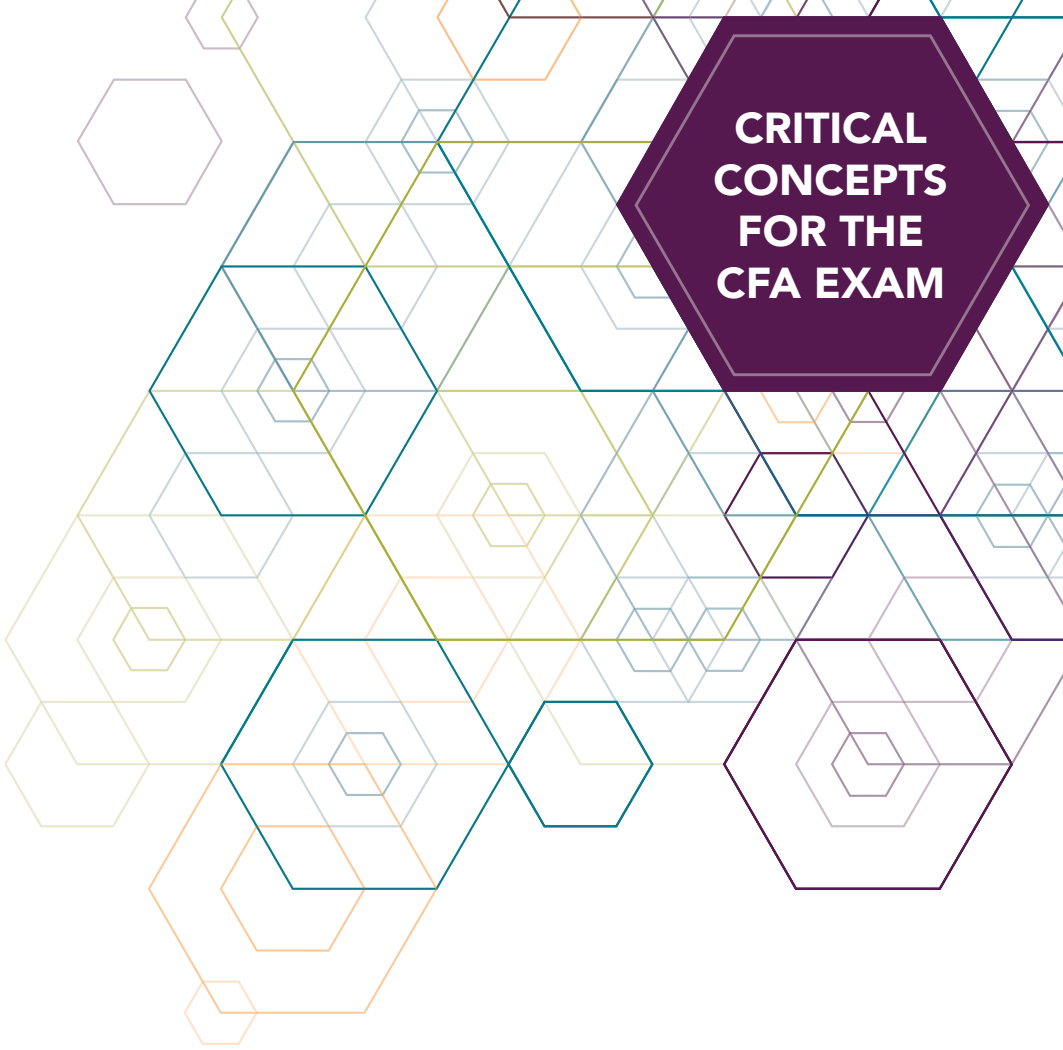
2020
CFA® EXAM REVIEW



WILEY'S CFA® PROGRAM LEVEL II SMARTSHEETS

FUNDAMENTALS FOR CFA EXAM SUCCESS

WILEY

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Wiley's CFA® Program Exam Review

ETHICAL AND PROFESSIONAL STANDARDS

STANDARDS OF PROFESSIONAL CONDUCT

I. Professionalism
   - A. Knowledge of the Law
   - B. Independence and Objectivity
   - C. Misrepresentation
   - D. Misconduct

II. Integrity of Capital Markets
   - A. Material Nonpublic Information
   - B. Market Manipulation

III. Duties to Clients
   - A. Loyalty, Prudence, and Care
   - B. Fair Dealing
   - C. Suitability
   - D. Performance Presentation
   - E. Preservation of Confidentiality

IV. Duties to Employers
   - A. Loyalty
   - B. Additional Compensation Arrangements
   - C. Responsibilities of Supervisors

V. Investment Analysis, Recommendations, and Actions
   - A. Diligence and Reasonable Basis
   - B. Communication with Clients and Prospective Clients
   - C. Record Retention

VI. Conflicts of Interests
   - A. Disclosure of Conflicts
   - B. Priority of Transactions
   - C. Referral Fees

VII. Responsibilities as a CFA Institute Member or CFA Candidate
   - A. Conduct as Participants in CFA Institute Programs
   - B. Reference to CFA Institute, the CFA Designation, and the CFA Program

QUANTITATIVE METHODS

LINEAR REGRESSION (1 INDEPENDENT VARIABLE)

- Standard error of the estimate (smaller SEE indicates better fit of regression model)

$$\text{SEE} = \left\{\frac{\sum_{i=1}^{n}(Y_i - \hat{b}_0 - \hat{b}_1 X_i)^2}{n-2}\right\}^{1/2} = \left\{\frac{\sum_{i=1}^{n}(\hat{\varepsilon}_i)^2}{n-2}\right\}^{1/2}$$

- Prediction interval around the predicted value of the dependent variable

$$s_f^2 = s^2\left[1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_x^2}\right]$$

$$\hat{Y} \pm t_c s_f$$

MULTIPLE REGRESSION (2 OR MORE INDEPENDENT VARIABLES)

- Confidence interval for regression coefficients: use $n - (k+1)$ degrees of freedom

$$\hat{b}_j \pm (t_c \times s_{\hat{b}_j})$$

$$\text{estimated regression coefficient} \pm (\text{critical } t\text{-value})(\text{coefficient standard error})$$

- Hypothesis test on each regression coefficient: use $n - (k+1)$ degrees of freedom

$$t\text{-stat} = \frac{\text{Estimated regression coefficient} - \text{Hypothesized value of regression coefficient}}{\text{Standard error of regression coefficient}}$$

- *p*-value: lowest level of significance at which we can reject the null hypothesis that the population value of the regression coefficient is zero in a two-tailed test (the smaller the *p*-value, the weaker the case for the null hypothesis)
- ANOVA table for testing whether all the slope coefficients are simultaneously equal to zero (use a one-tailed *F*-test and reject null hypothesis if *F*-statistic > F_{crit})

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F	Significance
Regression	k	RSS	MSR = RSS / k	MSR/MSE	p-value
Residual	n − (k + 1)	SSE	MSE = SSE /n − (k + 1)		
Total	n − 1	**SST**			

$$F\text{-stat} = \frac{\text{MSR}}{\text{MSE}} = \frac{\text{RSS}/k}{\text{SSE}/[n-(k+1)]}$$

- Standard error of the estimate (SEE) = √MSE using MSE from the ANOVA table
- Coefficient of determination (higher R^2 indicates a higher proportion of the total variation in dependent variable explained by the independent variables)

$$R^2 = \frac{\text{Total variation} - \text{Unexplained variation}}{\text{Total variation}} = \frac{\text{SST} - \text{SSE}}{\text{SST}} = \frac{\text{RSS}}{\text{SST}}$$

- Adjusted R^2

$$\text{Adjusted } R^2 = \bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2)$$

VIOLATIONS OF REGRESSION ASSUMPTIONS

- Heteroskedascity: variance of error term is not constant
  - Unconditional: heteroskedasticity is not related to the independent variables (does not affect statistical inference).
  - Conditional: heteroskedasticity is correlated with the independent variables (causes *F*-test for overall significance of the regression and *t*-test for the significance of each regression coefficient to become unreliable).
- Serial correlation: regression errors are correlated across observations (could be positive or negative and has same effect on statistical inference as conditional heteroskedasticity)
- Multicollinearity: two or more independent variables (or combinations of independent variables) are highly correlated
  - Makes regression coefficients inaccurate and *t*-test for the significance of each regression coefficient unreliable.
  - Difficult to isolate the impact of each independent variable on the dependent variable.
- Model specification errors
  - Misspecified functional form (omitting important variables; variables may need to be transformed; pooling data incorrectly).
  - Time-series misspecification (including lagged dependent variables as independent variables in regressions when there is serial correlation of errors; including an independent variable that is a function of the dependent variable; measuring independent variables with error; nonstationarity).

TIME SERIES ANALYSIS

- Linear trend model: predicts that the dependent variable grows by a constant amount in each period

$$y_t = b_0 + b_1 t + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

- Log-linear trend model: predicts that the dependent variable exhibits exponential growth

$$\ln y_t = b_0 + b_1 t + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

- Autoregressive (AR) time series model: uses past values of the dependent variable to predict its current value
  - First-order AR model

$$x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$$

  - AR model must be covariance stationary and specified such that the error terms do not exhibit serial correlation and heteroskedasticity in order to be used for statistical inference.
  - *t*-test for serial (auto) correlation of the error terms (model is correctly specified if all the error autocorrelations are not significantly different from 0)

$$t\text{-stat} = \frac{\text{Residual autocorrelation for lag}}{\text{Standard error of residual autocorrelation}}$$

  - Mean-reverting level of AR(1) model

$$\text{Mean reverting level} = x_t = \frac{b_0}{1 - b_1}$$

  - Random walk is a special AR(1) model that is not covariance stationary (undefined mean reverting level)

$$x_t = x_{t-1} + \varepsilon_t, \mathrm{E}(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t^2) = \sigma^2, \mathrm{E}(\varepsilon_t \varepsilon_s) = 0 \text{ if } t \neq s$$

  - First difference of a random walk in order to make it covariance stationary (mean reverting level of 0)

$$y_t = x_t - x_{t-1} = x_{t-1} + \varepsilon_t - x_{t-1} = \varepsilon_t, \mathrm{E}(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t^2) = \sigma^2, \mathrm{E}(\varepsilon_t \varepsilon_s) = 0 \text{ for } t \neq s$$

  - AR(1) model has a unit root if the slope coefficient equals 1, e.g., a random walk.
  - Dickey-Fuller test indicates that a time series has a unit root if the null hypothesis is not rejected.
  - Seasonality in AR models: the seasonal autocorrelation of the error term will be significantly different from 0 (can be solved by introducing a seasonal lag in the model).
- ARCH models: used to determine whether the variance of the error in one period depends on the variance of the error in previous periods (if ARCH errors are found, use generalized least squares to correct for heteroskedasticity)
- Regression with two time series: use the Dickey-Fuller test to determine whether the independent variable and the dependent variable have a unit root
  - If neither of the time series has a unit root, linear regression can be used to test the relationships between the two time series.
  - If either of them has a unit root, linear regression cannot be used as results may be spurious.
  - If both of them have unit roots and if they are cointegrated, the regression coefficients and standard errors will be consistent and they can be used to conduct hypothesis tests.

MACHINE LEARNING

- Supervised ML algorithms include:
  - Penalized regression are models with a large number of independent variables can be simplified by removing the least influential variables
  - Support vehicle machine (SVM) which helps to classify/ segregate observations into distinct groups
  - K-nearest neighbour (KNN) is generally used to classify an new observation into an existing group based on data factors.
  - Classification and Regression Tree (CART) is a decision tree which can be use to visually classify observations into groups at terminal nodes.
  - Ensemble learning combines the predictions of multiple ML algorithms to classify observations. This may be different types of models run once (voting), or the same type of model run numerous times (bagging)
  - Random forest classifier uses the bagging method by using a large number of decision trees

- Unsupervised ML algorithms include:
  - Principal component analysis is used to reduce the number of independent variables by combining variables into composite variables that are uncorrelated with each other
  - Clustering is a group of algorithms that seperate observations into like groups. This includes k-means clustering (which groups observations into k groups) and heirarchal clustering (which breaks doen observations into smaller and smaller sub-groups.
- Deep learning ML algorithms include:
  - Neural networks which take a number of inputs and produces an output by working through relationship between input/output layers and nodes in a hidden layer
  - Deep learning net which are neural networks with multiple hidden layers
  - Reinforcement learning is conducted whereby the underlying algorithm is not fed labeled data or given instantaneous feedback, but instead learns from trial and error by observing its environment and testing actions to observe consequences

BIG DATA

- Accuracy = (TP +TN)/(TP + FP +TN +FN)
- Precision (P) = TP/(TP + FP)
- Recall (R) = TP/(TP + FN)
- F1 score = (2 x P x R)/(P + R)

  Where:

  TP = True positives

  TN = True negatives

  FP = False positives (Type I error)

  FN = False negatives (Type II error)

RISK TYPES AND PROBABILISTIC APPROACHES

Discrete/ Continuous	Correlated/ Independent	Sequential/ Concurrent	Risk Approach
Discrete	Independent	Sequential	Decision tree %
Discrete	Correlated	Concurrent	Scenario analysis
Continuous	Either	Either	Simulations

ECONOMICS

CURRENCY EXCHANGE RATES

- Exchange rates are expressed using the convention *p/b*, i.e. number of units of currency *p* (price currency) required to purchase one unit of currency *b* (base currency). USD/GBP = 1.5125 means that it will take 1.5125 USD to purchase 1 GBP
- Exchange rates with bid and ask prices
  - For exchange rate *p/b*, the bid price is the price at which the client can sell currency *b* (base currency) to the dealer. The ask price is the price at which the client can buy currency *b* from the dealer.
  - The *b/p* ask price is the reciprocal of the *p/b* bid price.
  - The *b/p* bid price is the reciprocal of the *p/b* ask price.
- Cross-rates with bid and ask prices
  - Bring the bid–ask quotes for the exchange rates into a format such that the common (or third) currency cancels out if we multiply the exchange rates

$$\frac{\text{JPY}}{\text{EUR}} = \frac{\text{JPY}}{\text{USD}} \times \frac{\text{USD}}{\text{EUR}}$$

  - Multiply bid prices to obtain the cross-rate bid price.
  - Multiply ask prices to obtain the cross-rate ask price.
  - Triangular arbitrage is possible if the dealer's cross-rate bid (ask) price is above (below) the interbank market's implied cross-rate ask (bid) price.
- Marking to market a position on a currency forward
  - Create an equal offsetting forward position to the initial forward position.
  - Determine the all-in forward rate for the offsetting forward contract.
  - Calculate the profit/loss on the net position as of the settlement date.
  - Calculate the PV of the profit/loss.
- Covered interest rate parity: currency with the higher risk-free rate will trade at a forward discount

$$F_{PC/BC} = S_{PC/BC} \times \frac{1 + (i_{PC} \times \text{Actual}/360)}{1 + (i_{BC} \times \text{Actual}/360)}$$

$$\text{Forward premium (discount) as a \%} = \frac{F_{PC/BC} - S_{PC/BC}}{S_{PC/BC}}$$

- Uncovered interest rate parity: expected appreciation/ depreciation of the currency offsets the yield differential

$$S^{e}_{FC/DC} = S_{FC/DC} \times \frac{(1 + i_{FC})}{(1 + i_{DC})}$$

- Relative purchasing power parity: high inflation leads to currency depreciation

$$\text{Relative PPP: } E(S^{T}_{FC/DC}) = S^{0}_{FC/DC}\left(\frac{1 + \pi_{FC}}{1 + \pi_{DC}}\right)^{T}$$

- Fisher and international Fisher effects: if there is real interest rate parity, the foreign-domestic nominal yield spread will be determined by the foreign-domestic expected inflation rate differential

$$\text{Fischer Effect: } i = r + \pi^{e}$$
$$\text{International Fisher effect: } (i_{FC} - i_{DC}) = (\pi^{e}_{FC} - \pi^{e}_{DC})$$

- FX carry trade: taking long positions in high-yield currencies and short positions in low-yield currencies (return distribution is peaked around the mean with negative skew and fat tails)
- Mundell-Fleming model with high capital mobility
  - A restrictive (expansionary) monetary policy under floating exchange rates will result in appreciation (depreciation) of the domestic currency.
  - A restrictive (expansionary) fiscal policy under floating exchange rates will result in depreciation (appreciation) of the domestic currency.
  - If monetary and fiscal policies are both restrictive or both expansionary, the overall impact on the exchange rate will be unclear.
- Mundell-Fleming model with low capital mobility (trade flows dominate)
  - A restrictive (expansionary) monetary policy will lower (increase) aggregate demand, resulting in an increase (decrease) in net exports. This will cause the domestic currency to appreciate (depreciate).
  - A restrictive (expansionary) fiscal policy will lower (increase) aggregate demand, resulting in an increase (decrease) in net exports. This will cause the domestic currency to appreciate (depreciate).
  - If monetary and fiscal stances are not the same, the overall impact on the exchange rate will be unclear.
- Monetary models of exchange rate determination (assumes output is fixed)
  - Monetary approach: higher inflation due to a relative increase in domestic money supply will lead to depreciation of the domestic currency.
  - Dornbusch overshooting model: in the short run, an increase in domestic money supply will lead to higher inflation and the domestic currency will decline to a level lower than its PPP value; in the long run, as domestic interest rates rise, the nominal exchange rate will recover and approach its PPP value.

ECONOMIC GROWTH

- Growth accounting equation (based on Cobb-Douglas production function)

$$\Delta Y/Y = \Delta A/A + \alpha\Delta K/K + (1 - \alpha)\Delta L/L$$

- Labor productivity growth accounting equation

$$\text{Growth rate in potential GDP} = \text{Long-term growth rate of labor force} + \text{Long-term growth rate in labor productivity}$$

- Classical growth model (Malthusian model)
  - Growth in real GDP per capita is temporary: once it rises above the subsistence level, it falls due to a population explosion.
  - In the long run, new technologies result in a larger (but not richer) population.
- Neoclassical growth model (Solow's model)
  - Both labor and capital are variable factors of production and suffer from diminishing marginal productivity.
  - In the steady state, both capital per worker and output per worker are growing at the same rate, $\theta/(1 - \alpha)$, where θ is the growth rate of total factor productivity and α is the elasticity of output with respect to capital.
  - Marginal product of capital is constant and equal to the real interest rate.
  - Capital deepening has no effect on the growth rate of output in the steady state, which is growing at a rate of $\theta/(1 - \alpha) + n$, where n is the labor supply growth rate.
- Endogenous growth model
  - Capital is broadened to include human and knowledge capital and R&D.
  - R&D results in increasing returns to scale across the entire economy.
  - Saving and investment can generate self-sustaining growth at a permanently higher rate as the positive externalities associated with R&D prevent diminishing marginal returns to capital.
- Convergence
  - Absolute: regardless of their particular characteristics, output per capita in developing countries will eventually converge to the level of developed countries.
  - Conditional: convergence in output per capita is dependent upon countries having the same savings rates, population growth rates and production functions.
  - Convergence should occur more quickly for an open economy.

ECONOMICS OF REGULATION

- Economic rationale for regulatory intervention: informational frictions (resulting in adverse selection and moral hazard) and externalities (free-rider problem)
- Regulatory interdependencies: regulatory capture, regulatory competition, regulatory arbitrage
- Regulatory tools: price mechanisms (taxes and subsidies), regulatory mandates/restrictions on behaviors, provision of public goods/financing for private projects
- Costs of regulation: regulatory burden and net regulatory burden (private costs – private benefits)
- Sunset provisions: regulators must conduct a new cost-benefit analysis before regulation is renewed

FINANCIAL REPORTING AND ANALYSIS

INTERCORPORATE INVESTMENTS

- Investments in financial assets (usually < 20% interest) under IAS 39
  - Held-to-maturity (debt securities): reported at amortized cost using the effective interest method; interest income and realized gains/losses are

recognized in income statement.
- Fair value through profit or loss (held for trading and investments designated at fair value): initially recognized at fair value, then remeasured at fair value with unrealized and realized gains/losses, interest income and dividend income reported in income statement.
- Available-for-sale (AFS): initially recognized at fair value, then remeasured at fair value with unrealized gains/losses recognized in equity (other comp. income) while realized gains/losses, interest income and dividend income are recognized in income statement.
- Difference between IFRS and US GAAP: unrealized gains/losses on AFS debt securities arising from exchange rate movements are recognized in income statement under IFRS (other comp income under US GAAP).

- Investments in financial assets under IFRS 9
  - All financial assets are initially measured at fair value.
  - Debt instruments are subsequently measured at amortized cost, fair value through other comp income (FVOCI) or fair value through profit or loss (FVPL).
  - Equity investments held for trading must be measured at FVPL; other equity investments can be measured at FVPL or FVOCI.
- Investments in associates (20-50% interest, significant influence): use equity method
  - Investment is initially recognized on the investor's balance sheet at cost (within a single line item); investor's proportionate share of investee earnings (less dividends) increases carrying amount of investment.
  - Investor's proportionate share of investee earnings is reported within a single item in income statement.
  - Excess of purchase price over book value (if any) is first allocated to specific assets whose fair value exceeds book value: excess related to inventory is expensed while excess related to PP&E is depreciated over an appropriate period of time (investor adjusts carrying amount of investment on its balance sheet by reducing its share of investee profits in the income statement) and any remaining amount is treated as goodwill (not amortized but subject to annual impairment test).
  - Fair value option: unrealized gains/losses arising from changes in fair value as well as interest and dividends received are included in the investor's income.
- Joint ventures (shared control): use equity method
- Business combinations (controlling interest): use acquisition method
  - All assets (at fair value), liabilities (at fair value), revenues and expenses of acquiree are combined with those of parent/acquirer.
  - Transactions between acquirer and acquiree are eliminated.
  - Acquiree's equity accounts are ignored.
  - If acquirer owns less than 100% equity interest in acquiree, it must create a non-controlling interest account on consolidated balance sheet and income statement to reflect proportionate share in acquiree's net assets and net income that belongs to minority shareholders.
  - Full goodwill method: goodwill equals the excess of total fair value of acquiree over fair value of its identifiable net assets.
  - Partial goodwill method: goodwill equals the excess of purchase price over fair value of the acquirer's proportionate share of acquiree's identifiable net assets.
  - Goodwill is not amortized but subject to annual impairment test.
  - Difference between IFRS and US GAAP: IFRS permits full and partial goodwill methods (US GAAP requires use of full goodwill method).
  - Impact of different accounting methods on financial ratios

	Equity Method	Acquisition Method
Leverage	Better (lower) as liabilities are lower and equity is the same	Worse (higher) as liabilities are higher and equity is the same
Net Profit Margin	Better (higher) as sales are lower and net income is the same	Worse (lower) as sales are higher and net income is the same
ROE	Better (higher) as equity is lower and net income is the same	Worse (lower) as equity is higher and net income is the same
ROA	Better (higher) as net income is the same and assets are lower	Worse (lower) as net income is the same and assets are higher

ACCOUNTING FOR DEFINED BENEFIT PENSION PLANS

- Pension obligation components

Pension obligation at the beginning of the period
+ Current service costs
+ Interest costs
+ Past service costs
+ Actuarial losses
– Actuarial gains
– Benefits paid
Pension obligation at the end of the period

- Fair value of plan assets

Fair value of plan assets at the beginning of the period
+ Actual return on plan assets
+ Contributions made by the employer to the plan
– Benefits paid to employees
Fair value of plan assets at the end of the period

- Balance sheet liability (or asset) equals funded status
  - Negative funded status = plan is underfunded = net pension liability.
  - Positive funded status = plan is overfunded = net pension asset.

$$\text{Funded status} = \text{Fair value of plan assets} - \text{Pension obligation}$$

- Periodic pension cost calculation (same for IFRS and US GAAP)

$$\text{Period c p nsion cost} = \text{End ngne t p nsion liab lity} - \text{Beg nning net p nsion liab lity} + \text{Emp oyer contrib t ions}$$

$$\text{Periodic pension cost} = \text{Current service costs} + \text{Interest costs} + \text{Past service costs} + \text{Actuarial losses} - \text{Actuarial gains} - \text{Actual return on plan assets}$$

- Periodic pension cost reported in P&L (also known as periodic pension expense)
  - IFRS: current service costs, past service costs and net interest expense/income recognized in P&L (remeasurement refers to items in OCI).
  - US GAAP: current service costs, interest expense, expected return on plan assets, amortization of past service costs and amortization of actuarial gains and losses recognized in P&L (past service costs and actuarial gains/losses are usually recognized in OCI before subsequent amortization to P&L).
- Impact of key assumptions on net pension liability and periodic pension cost

Assumption	Impact of Assumption on Net Pension Liability (Asset)	Impact of Assumption on Periodic Pension Cost and Pension Expense
Higher discount rate	Lower obligation	Pension cost and pension expense will both typically be lower because of lower opening obligation and lower service costs.
Higher rate of compensation increase	Higher obligation	Higher service and interest costs will increase periodic pension cost and pension expense.
Higher expected return on plan assets	No effect, because fair value of plan assets are used on balance sheet	Not applicable for IFRS. No effect on periodic pension cost under U.S. GAAP. Lower periodic pension expense under U.S. GAAP.

MULTINATIONAL OPERATIONS

- For independent subsidiary
  - Local currency (LC) = functional currency (FC) ≠ parent's presentation currency (PC).
  - Use current rate method to translate accounts from LC to PC.
  - Income statement at average rate.
  - Assets and liabilities at current rate.
  - Capital stock at historical rate.
  - Dividends at rate when declared.
  - Translation gain/loss included in equity under cumulative translation adjustment (CTA).
  - Exposure = net assets.
- For well-integrated subsidiary
  - LC ≠ FC = PC.
  - Use temporal method to translate accounts from LC to PC.
  - Monetary assets and liabilities at current rate.
  - Nonmonetary assets and liabilities at historical rate.
  - Capital stock at historical rate.
  - Revenues and expenses at average rate, except for expenses related to nonmonetary assets (e.g. COGS, depreciation) which are translated at historical rates.
  - Dividends at rate when declared.
  - Translation gain/loss reported in income statement.
  - Exposure = net monetary asset or liability.
- Net asset (liability) exposure and appreciating foreign currency = translation gain (loss)
- Ratios (originally in LC versus current rate method)
  - Pure income statement and balance sheet ratios unaffected.
  - If foreign currency is appreciating (depreciating), mixed ratios (based on year-end b/sheet values) will be smaller (larger) after translation.
- Hyperinflationary economies
  - US GAAP: use temporal method.
  - IFRS: (1) restate subsidiary's foreign currency accounts for inflation; (2) translate using current exchange rate; (3) gain/loss in purchasing power recorded on income statement.

EVALUATING QUALITY OF FINANCIAL REPORTS

- Beneish model: the higher the *M*-score (i.e. the less negative the number) the higher the probability of earnings manipulation
- Altman bankruptcy protection model: higher *z*-score is better

INTEGRATED FINANCIAL STATEMENT ANALYSIS

- ROE decomposition (extended DuPont analysis)

$$\text{ROE} = \text{Tax Burden} \times \text{Interest burden} \times \text{EBIT margin} \times \text{Total asset turnover} \times \text{Financial leverage}$$

$$\text{ROE} = \frac{\text{NI}}{\text{EBT}} \times \frac{\text{EBT}}{\text{EBIT}} \times \frac{\text{EBIT}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Average Asset}} \times \frac{\text{Average Asset}}{\text{Average Equity}}$$

CORPORATE FINANCE

CAPITAL BUDGETING

- Initial investment outlay
  - New investment

$$\text{Initial inv stment for a new inv stment} = \text{FCInv} + \text{NWCInv}$$

  - Replacement project

$$\text{Initial investment for a replacement project} = \text{FCInv} + \text{NWCInv} - \text{Sal}_0 + t(\text{Sal}_0 - \text{BV}_0)$$

- Annual after-tax operating cash flows (CF)

$$\text{CF} = (\text{S} - \text{C} - \text{D})(1 - t) + \text{D} \quad \text{or} \quad \text{CF} = (\text{S} - \text{C})(1 - t) + t\text{D}$$

- Terminal year after-tax non-operating cash flows (TNOCF)

$$\text{TNOCF} = \text{Sal}_\text{T} + \text{NWCInv} - t(\text{Sal}_\text{T} - \text{B}_\text{T})$$

- Inflation reduces the value of depreciation tax savings: if

inflation is higher (lower) than expected, the profitability of the project will be lower (higher) than expected
- Mutually exclusive projects with unequal lives
  - Least common multiple of lives approach: choose project with higher NPV.
  - Equivalent annual annuity (EAA) approach: choose project with higher EAA (annuity payment over the project's life with same NPV as project's NPV).
- Capital rationing: if budget is fixed, use NPV or profitability index (PI) to rank projects
- Project discount rate using CAPM

$$R_i = R_F + \beta_i[E(R_M) - R_F]$$

- Real options: timing, sizing (abandonment and expansion), flexibility, fundamental
- Economic income

Economic income = After-tax operating cash flow + Change in market value
Economic income = After-tax operating cash flow + (Ending market value − Beginning market value)

OR

Economic income = After-tax operating cash flow − (Beginning market value − Ending market value)

Economic income = After-tax cash flows − Economic depreciation

- Economic profit

Economic profit = [EBIT (1 − Tax rate)] − $WACC

Economic profit = NOPAT − $WACC

- Claims valuation
  - Separate cash flows available to debt and equity holders.
  - Discount them at their respective required rates of return (debt cash flows discounted at cost of debt, equity cash flows discounted at cost of equity).
  - Add PVs of the two cash flow streams to calculate total company/asset value.

CAPITAL STRUCTURE

- MM Prop I without taxes: given MM assumptions and no taxes, changes in capital structure do not affect company value
- MM Prop II without taxes: higher financial leverage raises the cost of equity but no change in WACC

$$r_E = r_0 + (r_0 - r_D)\frac{D}{E}$$

- MM Prop I with taxes: debt results in tax savings, so company value would be maximized with 100% debt (no costs of financial distress)
- MM Prop II with taxes: higher financial leverage raises the cost of equity and lowers WACC (WACC is minimized at 100% debt)

$$r_{WACC} = \left(\frac{D}{V}\right) r_D (1 - t) + \left(\frac{E}{V}\right) r_E$$

$$r_E = r_0 + (r_0 - r_D)(1 - t)\left(\frac{D}{E}\right)$$

- Agency costs: using more debt reduces net agency costs of equity
- Pecking order theory (information asymmetry): managers prefer internal financing and debt over equity
- Static trade-off theory (optimal capital structure): increase debt up to the point where further increases in value from tax savings are offset by additional costs of financial distress

DIVIDENDS AND SHARE REPURCHASES

- Dividend policy
  - MM: with perfect capital markets, dividend policy does not matter because shareholders can create homemade dividends.
  - Bird-in-hand argument: even with perfect capital markets, shareholders prefer current dividends over future capital gains.
  - Tax argument: if higher tax on dividends vs capital gains, investors prefer earnings reinvestment and share repurchases over cash dividends.
- Signaling effect: dividend initiations or increases usually taken as positive signals (unless overvalued company)
- Agency costs: shareholders prefer cash dividends to prevent managers investing in negative NPV projects; bondholders often restrict dividends through covenants
- Factors affecting dividend policy: investment opportunities, expected volatility of earnings, financial flexibility, tax considerations, flotation costs, contractual/legal restrictions
- Effective tax rate (ETR) when given corporate tax rate for earnings distributed as dividends (CTR_D) and investor's marginal tax rate on dividends (MTR_D)
  - Double taxation and split-rate

$$ETR = CTR_D + [(1 - CTR_D) \times MTR_D]$$

  - Imputation: $ETR = MTR_D$
- Payout policy
  - Stable dividend policy

Expected increase in dividends = (Expected earnings × Target payout ratio − Previous dividend) × Adjustment factor

  - Constant dividend payout ratio policy: payout is a constant % of net income.
  - Residual dividend policy: payout only if there is sufficient cash after investment in positive NPV projects.
- Share repurchases
  - All else being equal, impact of share repurchase on shareholder wealth is the same as that of cash dividends.
  - Reasons to prefer share repurchase: potential tax advantages, share price support, managerial flexibility, offset dilution from employee stock options, higher financial leverage.
- Effect of share repurchase on EPS
  - If funds used for share repurchase are generated internally, EPS will increase if the funds would not have earned the cost of capital if retained.
  - If borrowing used to finance share repurchase, EPS will fall (rise) if after-tax cost of borrowing is higher (lower) than earnings yield.
- Affect of share repurchase on book value per share (BVPS): when market price is higher (lower) than BVPS, BVPS will decrease (increase) after repurchase
- Dividend safety measure

FCFE coverage ratio = FCFE / [Dividends + Share repurchases]

CORPORATE GOVERNANCE

- Concentrated ownership structures (which are more prevalent on a global basis) can lead to large shareholders having more power over managers and minority shareholders than in a dispersed ownership structure.
- Effective boards might have a chairperson that is separate from the CEO, a two-tier structure and a majority of independent directors.
- Environmental factors for an anlalyst to assess might include energy efficiency and product packaging
- Social factors might include customer welfare and marketing practices
- Governance factors might include board structure and competitive behavior.

MERGERS AND ACQUISITIONS

- Mergers and industry lifecycle
  - Pioneering development: conglomerate and horizontal.
  - Rapid accelerating growth: conglomerate and horizontal.
  - Mature growth: horizontal and vertical.
  - Stabilization and market maturity: horizontal.
  - Deceleration of growth and decline: horizontal, vertical and conglomerate.
- Pre-offer takeover defense mechanisms: poison pills, poison puts, incorporation in a state with restrictive laws, staggered board of directors, restricted voting rights, supermajority voting provisions, fair price amendments, golden parachutes
- Post-offer takeover defense mechanisms: litigation, greenmail, share repurchase, leveraged recapitalization, "just say no," "crown jewel," "Pac–man," white knight and white squire defenses
- Herfindahl-Hirschman Index (HHI)

$$\sum_i^n \left(\frac{\text{Sales or output of firm i}}{\text{Total sales or output of market}} \times 100\right)^2$$

Post-Merger HHI	Concentration	Change in HHI	Government Action
Less than 1,000	Not concentrated	Any amount	No action
Between 1,000 and 1,800	Moderately concentrated	100 or more	Possible challenge
More than 1,800	Highly concentrated	50 or more	Challenge

- Target company valuation
  - DCF analysis based on FCFF.
  - Comparable company analysis: relative valuation measures used to estimate market value of target, then add takeover premium.
  - Comparable transaction analysis: recent merger transactions used to estimate fair acquisition price for target (takeover premium built into transaction prices).
- Merger bid evaluation
  - Post-merger value of the combined company

$V_A^* = V_A + V_T + S - C$

V_A^* = Post-merger value of the combined company
V_A = Pre-merger value of the acquirer
V_T = Pre-merger value of the target company
S = Synergies created by the business combination
C = Cash paid to target shareholders

  - Takeover premium and acquirer's gain

Target shareholders' gain = Takeover premium = $P_T - V_T$

Acquirer's gain = Synergies − Premium
= $S - (P_T - V_T)$

S = Synergies created by the merger transaction

  - Acquirer prefers cash offer if confident of synergies and/or target's value.

EQUITY INVESTMENTS

EQUITY VALUATION MODELS

- Absolute valuation: estimate asset's intrinsic value, e.g., dividend discount model
- Relative valuation: estimate asset's value relative to that of another asset, e.g., price multiples

RETURN CONCEPTS

- Holding period return

$$\text{Holding period return} = \frac{P_H - P_0 + D_H}{P_0}$$

- Required return
  - Minimum level of return on an asset required by an investor.
  - If expected return is higher (lower) than required return, the asset is undervalued (overvalued).
- Equity risk premium (ERP)
  - Additional return required by investors to invest in equities rather than risk-free asset.
  - Gordon growth model estimate of ERP

$$ERP_{GGM} = \frac{D_1}{P_0} + g_e - Y_{LTGB}$$

- Supply-side estimate (Ibbotson-Chen) of ERP

Equity risk premium = [{(1 + EINFL) (1 + EGREPS) (1 + EGPE) – 1] + EINC} – Expected RF

- Estimating the required return on equity to discount cash flows to equity
  - CAPM

$$r_i = r_f + \beta_{i,M}(r_M - r_f)$$

  - Fama-French model

$$r_i = RF + \beta_i^{mkt} RMRF + \beta_i^{size} SMB + \beta_i^{value} HML$$

  - Pastor-Stambaugh model: adds a liquidity factor to the Fama-French model.
  - Macroeconomic multifactor models: use economic variables as factors.
  - Build-up method for private business

r_i = Risk-free rate + Equity risk premium + Size premium + Specific-company premium

  - Bond yield plus risk premium (BYPRP) approach with publicly-traded debt

BYPRP cost of equity = YTM on the company's long-term debt + Risk premium

- Adjusting beta for beta drift

Adjusted beta = (2/3) (Unadjusted beta) + (1/3) (1.0)

- Estimating beta for non-public company using the pure-play method

$$\beta_{ASSET} = \beta_{EQUITY}\left[\frac{1}{1+\left(\frac{D}{E}\right)}\right]$$

$$\beta_{PROJECT} = \beta_{ASSET}\left[1+\frac{D}{E}\right]$$

- Weighted average cost of capital (WACC) to discount cash flows to the firm

$$WACC = \frac{MVD}{MVD + MVCE} r_d (1 - \text{Tax rate}) + \frac{MVCE}{MVD + MVCE} r$$

INDUSTRY AND COMPANY ANALYSIS

- Projecting future sales growth
  - Growth relative to GDP growth approach

$$g_S = \beta_{S,GDP} \times g_{GDP}$$

  - Market growth and market share approach

$$g_S = (1 + g_M)(1 + g_{MS}) - 1$$

- Return measure
  - Return on invested capital (ROIC): better measure of profitability than ROE because unaffected by financial leverage

ROIC = NOPLAT / Invested capital

  - Return on capital employed (ROCE): pretax measure useful for comparisons across different countries/tax structures

ROCE = Operating profit / Capital employed

- Analysing competitive position with Porter's five forces
  - Threat of substitutes.
  - Rivalry (intensity of competition).
  - Bargaining power of suppliers.
  - Bargaining power of customers.
  - Threat of new entrants.

DISCOUNTED DIVIDEND VALUATION

- Use dividends as a measure of cash flow when:
  - Company has dividend history.
  - Dividend policy is related to earnings.
  - Non-control perspective.
- Gordon growth model: constant dividend growth to infinity

$$V_0 = \frac{D_0(1+g)}{(r-g)}, \text{ or } V_0 = \frac{D_1}{(r-g)}$$

- Present value of growth opportunities (PVGO)

$$V_0 = \frac{E_1}{r} + PVGO$$

- Two-stage DDM: high growth rate in the short run (first stage), lower growth rate in long run (second stage)

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_S)^t}{(1+r)^t} + \frac{D_0(1+g_S)^n(1+g_L)}{(1+r)^n(r-g_L)}$$

- H-model: growth rate declines linearly from a short-run high rate to long-run constant growth rate (H = half the length of the high growth period)

$$V_0 = \frac{D_0(1+g_L)}{r-g_L} + \frac{D_0 H(g_S - g_L)}{r-g_L}$$

- Sustainable growth rate

$$g = b \times ROE$$

b = Earnings retention rate, calculated as 1 – Dividend payout ratio

$$ROE = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Shareholders' equity}}$$
$$= \text{Profit margin} \times \text{Asset turnover} \times \text{Financial leverage}$$

FREE CASH FLOW

- Use free cash flow for valuation when:
  - Company does not pay dividends or pays dividends that deviate significantly from FCFE.
  - Free cash flow is related to profitability.
  - Investor takes a control perspective.
- Free cash flow to the firm (FCFF)

$$FCFF = NI + NCC + Int(1 - \text{Tax Rate}) - FCInv - WCInv$$

$$FCFF = EBIT(1 - \text{Tax rate}) + Dep - FCInv - WCInv$$

- Free cash flow to equity (FCFE)

$$FCFE = FCFF - Int(1 - \text{Tax rate}) + \text{Net borrowing}$$

$$FCFE = NI + NCC - FCInv - WCInv + \text{Net Borrowing}$$

$$FCFE = EBIT(1 - \text{Tax rate}) - Int(1 - \text{Tax rate}) + Dep - FCInv - WCInv + \text{Net borrowing}$$

- FCFE is simpler to use when capital structure is stable
- FCFF is preferred if it reflects company fundamentals better or if FCFE is negative
- Single-stage FCFF/FCFE valuation model

$$\text{Value of the firm} = \frac{FCFF_1}{WACC - g}$$

$$\text{Value of equity} = \frac{FCFE_1}{r-g}$$

- Two-stage FCFF/FCFE valuation model

$$\text{Firm value} = \sum_{t=1}^{n} \frac{FCFF_t}{(1+WACC)^t} + \frac{FCFF_{n+1}}{(WACC-g)} \frac{1}{(1+WACC)^n}$$

Firm value = PV of FCFF in Stage 1 + Terminal value × Discount Factor

$$\text{Equity value} = \sum_{t=1}^{n} \frac{FCFE_t}{(1+r)^t} + \frac{FCFE_{n+1}}{r-g} \frac{1}{(1+r)^n}$$

Equity value = PV of FCFE in Stage 1 + Terminal value × Discount Factor

PRICE AND ENTERPRISE VALUE MULTIPLES

- Price to earnings (P/E) ratio
  - Earnings are a key driver of stock value but could be negative.
  - May be difficult to identify recurring earnings.
  - Affected by accounting choices.
- Normalizing earnings for a cyclical company
  - Historical average EPS (does not account for changes in company size).
  - Average ROE (accounts for changes in company size).
- Justified P/E

$$\text{Justified leading P/E} = \frac{P_0}{E_1} = \frac{D_1/E_1}{r-g} = \frac{(1-b)}{r-g}$$

$$\text{Justified trailing P/E} = \frac{P_0}{E_0} = \frac{D_1/E_0}{r-g} = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g}$$

- P/E-to-growth (PEG) ratio: investors prefer stocks with lower PEGs
  - PEG ratio assumes linear relationship between P/E and growth.
  - Does not account for different risk and duration of growth.
- Price to book value (P/B) ratio
  - Book value usually positive and more stable than earnings.
  - Useful for financial sector companies with liquid assets.
  - Misleading when there are non-tangible factors and size differences.
  - Affected by accounting choices.
  - Inflation/technology may cause big differences between BV and MV.
- Justified P/B

$$\frac{P_0}{B_0} = \frac{ROE - g}{r-g}$$

- Price to sales (P/S) ratio
  - Sales less affected by accounting choices than earnings and book value.
  - Sales positive even when earnings are negative and more stable than earnings.
  - Useful for mature, cyclical and loss-making companies.
  - Sales ≠ profits and does not reflect cost structure.
  - Sales may be distorted due to revenue recognition choices.
- Justified P/S

$$\frac{P_0}{S_0} = \frac{(E_0/S_0)(1-b)(1+g)}{r-g}$$

- Price to cash flow (P/CF) ratio
  - Cash flow less affected by accounting choices than earnings.
  - Cash flow more stable than earnings.
  - Many definitions of cash flow.
- Enterprise value to EBITDA multiple
  - Useful for comparing companies with different leverage.
  - Useful for valuing capital-intensive firms.
  - EBITDA is often positive when earnings are negative.
  - EBITDA is affected by revenue recognition choices.
  - Enterprise value = MV of common equity + MV of preferred stock + MV of debt – Value of cash and short-term investments.
- Weighted harmonic mean for portfolio P/E

$$\text{Weighted harmonic mean} = X_{WH} = \frac{1}{\sum_{i=1}^{n}(w_i / X_i)}$$

RESIDUAL INCOME

- Use residual income (RI) for valuation when:
  - Company does not pay dividends.
  - Free cash flow expected to be negative.
  - Accounting disclosures are good.
- RI model is not appropriate when:
  - Clean surplus relation is violated.
  - Book value and ROE are difficult to predict.
- RI calculation

$$RI_t = E_t - (r \times B_{t-1})$$

$$RI_t = (ROE_t - r)B_{t-1}$$

- Single-stage RI model

$$V_0 = B_0 + \frac{ROE - r}{r - g}B_0$$

- Multi-stage RI model

$$V_0 = B_0 + (\text{PV of future RI over the short-term}) + (\text{PV of continuing RI})$$

$$V_0 = B_0 + \sum_{t=1}^{T}\frac{(ROE_t - r)B_{t-1}}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$$

$$V_0 = B_0 + \sum_{t=1}^{T-1}\frac{(E_t - rB_{t-1})}{(1+r)^t} + \frac{E_T - rB_{T-1}}{(1+r-\omega)(1+r)^{T-1}}$$

- Economic Value Added (EVA)

$$EVA = [EBIT\ (1 - \text{Tax rate})] - (C\% \times TC)$$

$$EVA = NOPAT - \$WACC$$

PRIVATE COMPANY VALUATION

- Income approach (suitable for companies experiencing high growth)
  - Free cash flow method.
  - Capitalized cash flow method (capitalization rate is discount rate minus growth rate).
  - Excess earnings method (calculates firm value by adding value of intangible assets to working capital and fixed assets).
- Market approach (use for stable, mature companies)
  - Guideline public company method (based on minority interest).
  - Guideline transaction method (based on control perspective).
  - Prior transaction method (usually based on minority interest).
- Asset-based approach (use for start-ups, firms with minimal profits, banks, REITs, natural resources)
- Discount for lack of control (DLOC)

$$DLOC = 1 - \left[\frac{1}{1 + \text{Control premium}}\right]$$

- Total discount with DLOC and discount for lack of marketability (DLOM)

$$\text{Total discount} = 1 - [(1 - DLOC)(1 - DLOM)]$$

FIXED INCOME

TERM STRUCTURE

- Forward pricing model

$$P(T^* + T) = P(T^*)F(T^*, T)$$

- Forward rate model

$$[1 + r(T^* + T)]^{T^*+T} = [1 + r(T^*)]^{T^*}[1 + f(T^*, T)]^T$$

$$r(T^* + T) = \left\{[1 + r(T^*)]^{T^*}[1 + f(T^*, T)]^T\right\}^{1/(T^*+T)} - 1$$

- Riding the yield curve: Assuming an upward-sloping, stable yield curve, a trader can enhance yield by buying bonds with maturity greater than rather than equal to the liability horizon.
- Swap spread = Swap fixed rate – Yield on government security with equivalent maturity
- z-spread = constant spread that is added to implied spot curve such that the PV of a bond's cash flows (when discounted at relevant spot rates plus the z-spread) equals its market price
- TED spread = LIBOR – Yield on a T-bill with same maturity
- LIBOR-OIS spread = LIBOR – overnight indexed swap rate
- Traditional theories of term structure
  - Unbiased (pure) expectations theory.
  - Local expectations theory.
  - Liquidity preference theory.
  - Segmented markets theory.
  - Preferred habitat theory.
- Modern term structure models
  - Cox-Ingersoll-Ross: short-term rate determines the entire term structure, interest rates are mean-reverting, volatility proportional to short-term rate, no negative interest rates.
  - Vasicek: short-term rate determines the entire term structure, interest rates are mean-reverting, volatility constant, negative interest rates possible.
  - Ho-Lee: arbitrage-free model, drift term is inferred from market prices so that the model can accurately generate the current term structure, volatility can be modeled as a function of time, negative interest rates possible.
- Yield curve risk can be managed using:
  - Key rate duration.
  - A measure based on a factor model which explains changes in the yield curve through level, steepness and curvature movements.
- Term structure of interest rate volatilities
  - Measure of yield curve risk.
  - Short-term rates usually more volatile than long-term rates.

ARBITRAGE-FREE VALUATION

- Use binomial interest rate tree and backward induction for option-free bonds and bonds with embedded options (except where bond's cash flows are interest rate path-dependent)
- Use Monte Carlo method to simulate a large number of potential interest rate paths in order to value a bond whose cash flows are interest rate path-dependent

BONDS WITH EMBEDDED OPTIONS

- Callable bond

$$\text{Value of callable bond} = \text{Value of straight bond} - \text{Value of embedded call option}$$

- Putable bond

$$\text{Value of putable bond} = \text{Value of straight bond} + \text{Value of embedded put option}$$

- Effect of interest rate volatility
  - Higher interest rate vol. increases value of embedded call option and decreases value of callable bond.
  - Higher interest rate vol. increases value of embedded put option and increases value putable bond.
- Effect of yield curve change: value of embedded call (put) option increases (decreases) as yield curve goes from upward sloping to flat to downward sloping
- Valuation of callable and putable bonds with binomial interest rate tree
  - Callable bond: at each node during the call period, the value of the bond must equal the lower of (1) the value if the bond is not called (using the backward induction), and (2) the call price.
  - Putable bond: at each node we use the higher of (1) the value determined through backward induction, and (2) the put price.
- Option-adjusted spread (OAS)
  - Constant spread that, when added to all one-year forward rates in interest rate tree, makes arbitrage-free value of bond equal to its current market price.
  - If the OAS for a bond is lower (higher) than that for a bond with similar characteristics and credit quality, it suggests that the bond is relatively overpriced (underpriced).
  - For a given bond price, the lower the interest rate volatility, the higher the OAS for a callable bond.
- Effective duration

$$\text{Effective Duration} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{ Curve}) \times PV_0}$$

Type of Bond	Effective Duration
Cash	0
Zero-coupon bond	≈ Maturity
Fixed-rate bond	< Maturity
Callable bond	≤ Duration of straight bond
Putable bond	≤ Duration of straight bond
Floater (Libor flat)	≈ Time (in years) to next reset

- Effective convexity
  - Callable bond: when interest rates fall and the embedded call option is at the money, effective convexity turns negative because the bond's price is capped at the call price.
  - Putable bond: when interest rates rise and the embedded put option is at the money, effective convexity remains positive but the downside is limited by the put price.
- Floaters

$$\text{Value of capped floater} = \text{Value of uncapped floater} - \text{Value of embedded cap}$$

$$\text{Value of floored floater} = \text{Value of non-floored floater} + \text{Value of embedded floor}$$

- Convertible bonds

$$\text{Conversion value} = \text{Market price of common stock} \times \text{Conversion ratio}$$

$$\text{Market conversion price} = \frac{\text{Market price of convertible security}}{\text{Conversion ratio}}$$

$$\text{Market conversion premium per share} = \text{Market conversion price} - \text{Current market price}$$

$$\text{Market conversion premium ratio} = \frac{\text{Market conversion premium per share}}{\text{Market price of common stock}}$$

$$\text{Premium over straight value} = \frac{\text{Market price of convertible bond}}{\text{Straight value}} - 1$$

$$\text{Minimum value} = \text{greater of conversion value or straight value}$$

$$\text{Convertible callable and putable bond value} = \text{Straight value} + \text{Value of the call option on the stock} - \text{Value of the call option on the bond} + \text{Value of the put option on the bond}$$

CREDIT ANALYSIS

- Loss given default = % of overall position lost if default occurs
- Recovery rate = % of overall position recovered if default occurs
- Expected loss = Probability of default × loss given default
- PV of expected loss = Difference between value of risky bond and value of equivalent riskless bond
- Structural models (option analogy)
  - Equity holders: comparable to holding a European call option on company assets.
  - Debt holders: comparable to holding a riskless bond and selling a European put option on company assets.
  - Model assumes that company assets trade in

frictionless markets.
- Structure of the balance sheet used to derive the model is unrealistic.
- Only implicit estimation can be used to estimate measures of credit risk because company asset value is an unobservable parameter.
- Credit risk measures do not explicitly consider changes in the business cycle.

- Reduced form models
  - Model assumes that only some of company's debt is traded.
  - Model inputs are observable, allowing the use of historical estimation for credit risk measures.
  - Credit risk measures consider changes in the business cycle.
  - Model does not impose any assumptions on balance sheet structure but needs to be properly formulated and backtested, e.g. hazard rate estimation.
- Credit analysis of ABS
  - Structural or reduced form model can be used.
  - ABS do not default, so probability of default replaced by probability of loss.

CREDIT DEFAULT SWAPS (CDS)

- Protection seller earns CDS spread and compensates protection buyer for credit losses if a credit event occurs
- Types of CDS: single-name CDS, index CDS, tranches CDS
- Credit events: bankruptcy, failure to pay, restructuring
- Settlement protocols: physical or cash
- Upfront payment/premium

Upfront payment = Present value of protection leg – Present value of premium leg

Upfront premium % ≅ (Credit spread – Fixed coupon) × Duration of CDS

- Price of CDS

Price of CDS per 100 par =100 – Upfront premium %

- Change in CDS price for a given change in credit spread

% Change in CDS price = Change in spread in bps × Duration

- Long/short trade: sell protection (long CDS) on entity whose credit quality is expected to improve and buy protection (short CDS) on entity whose credit quality is expected to worsen
- Curve trade with upward-sloping credit curve: if credit curve is expected to steepen, buy protection (short CDS) on a long-term CDS and sell protection (long CDS) on a short-term CDS of the same entity
- Basis trade: profit from temporary difference between (1) credit spread on a bond, and (2) credit spread on a CDS on same reference obligation with the same term to maturity

DERIVATIVES

FORWARDS AND FUTURES

- Forward price assuming no carry costs or benefits

$$F_0(T) = S_0 (1 + r)^T$$

- Value of a forward contract during its life assuming no carry costs or benefits (long position)

$$V_t(T) = S_t - [F_0(T) / (1 + r)^{T-t}]$$

- Forward price when underlying has discrete cash flows

$$F_0(T) = (S_0 - \gamma_0 + \theta_0)(1 + r)^T$$
$$F_0(T) = S_0(1 + r)^T - (\gamma_0 - \theta_0)(1 + r)^T$$

- Forward price when underlying has cash flows (continuous compounding)

$$F_0(T) = S_0 e^{(r_c + \theta_c - \gamma_c)T}$$

- Value of a forward contract during its life when underlying has cash flows (long position)

$$V_t(T) = PV \text{ of differences in forward prices} = PV_{t,T}[F_t(T) - F_0(T)]$$

- Price of a FRA: forward rate starting at FRA expiration, given two LIBOR rates
- Value of a FRA prior to expiration
  - Calculate new implied forward rate based on current LIBOR rates.
  - Calculate interest savings based on this new forward rate vs FRA rate.
  - Discount these interest savings for a period equal to the number of days remaining until FRA expiration plus the number of days in the term of the underlying hypothetical loan (using appropriate LIBOR rate).
- Price of a bond futures contract when accrued interest is not included in the bond price quote (convert this price to the quoted futures price using bond's conversion factor)

$$F_0(T) = [B_0(T + Y) + AI_0 - PVCI_{0,T}] \times (1 + r)^T - AI_T$$

- Price of a currency forward

$$F_{0,PC/BC} = S_{0,PC/BC} \times \frac{(1 + r_{PC})^T}{(1 + r_{BC})^T}$$

$$F_{0,PC/BC} = S_{0,PC/BC} \times e^{(r_{PC} - r_{BC}) \times T}$$

- Value of a currency forward (long position)

$$V_t(T) = (F_{t,PC/BC} - F_{0,PC/BC}) / (1 + r_{PC})^{T-t}$$

SWAPS

- Price of a plain vanilla interest rate swap (swap fixed rate)

$$\text{Swap fixed rate} = \left[\frac{1 - B_0(N)}{B_0(1) + B_0(2) + B_0(3) + \ldots + B_0(N)}\right] \times 100$$

- Value of a plain vanilla interest rate swap

V = NA * (PSFR$_0$ – PSFR$_t$) * Sum of PV factors of remaining coupon payments as of t = t

where PSFR is the periodic swap fixed rate.

- Value of an equity swap
  - Pay-fixed, receive-return-on-equity swap

[(1 + Return on equity) * Notional amount] – PV of the remaining fixed-rate payments

  - Pay-floating, receive-return-on-equity swap

[(1 + Return on equity) * Notional amount] – PV (Next coupon payment + Par value)

  - Pay-return on one equity instrument, receive-return on another equity instrument swap

[(1 + Return on Index 2) * Notional amount] – [(1 + Return on Index 1) * Notional amount]

OPTIONS

- One-period binomial model for European stock options
  - No-arbitrage approach and expectations approach give same answer.
  - Hedge ratio for call and put options

$$h = \frac{c^+ - c^-}{S^+ - S^-} > 0,\ h = \frac{p^+ - p^-}{S^+ - S^-} < 0$$

  - Value of call and put options using the no-arbitrage approach

$$c = hS + PV(-hS^- + c^-) \text{ or } c = hS + PV(-hS^+ + c^+)$$

$$p = hS + PV(-hS^- + p^-) \text{ or } p = hS + PV(-hS^+ + p^+)$$

  - Value of call option with expectations approach (where π = risk-neutral probability of UP move)

$$c = \frac{\pi c^+ + (1-\pi)c^-}{(1+r)}$$

where:

$$\pi = \frac{(1 + r - d)}{(u - d)}$$

  - Use process to value a put option using the expectations approach.
- Two-period binomial model for European stock options
  - Use backward induction with the expectations approach.
  - Value of call and put options

$$c = PV[\pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--}]$$
$$p = PV[\pi^2 p^{++} + 2\pi(1-\pi)p^{+-} + (1-\pi)^2 p^{--}]$$

- American options
  - American call options on a non-dividend-paying stock will never be exercised early.
  - Early exercise of American call options on a dividend-paying stock and American put options on both dividend-paying and non-dividend-paying stocks may be optimal in some cases.
  - Black-Scholes-Merton model for European options on non-dividend-paying stock

$$c = SN(d_1) - e^{-rT}XN(d_2)$$
$$p = e^{-rT}XN(-d_2) - SN(-d_1)$$

- Swaptions: holder of a payer (receiver) swaption hopes that market swap fixed rate increases (decreases) before expiration of swaption
- Calculating the optimal number of hedging units for delta hedging

$$N_H = -\frac{\text{Portfolio delta}}{\text{Delta}_H}$$

- Estimating the value of an option using delta and gamma

$$\text{For calls: } \hat{c} - c \approx \text{Delta}_c(\hat{S} - S) + \frac{\text{Gamma}_c}{2}(\hat{S} - S)^2$$

$$\text{For puts: } \hat{p} - p \approx \text{Delta}_p(\hat{S} - S) + \frac{\text{Gamma}_p}{2}(\hat{S} - S)^2$$

ALTERNATIVE INVESTMENTS

PRIVATE REAL ESTATE INVESTMENTS

- Net operating income

Rental income at full occupancy
+ Other income (such as parking)
= Potential gross income (PGI)
– Vacancy and collection loss
= Effective gross income (EGI)
– Operating expenses (OE)
= Net operating income (NOI)

- Direct capitalization method
  - Capitalization rate from comparable property

Cap rate = Discount rate – Growth rate

  - Value of property

$$\text{Value} = \frac{NOI_1}{\text{Cap rate}}$$

- Gross income multiplier method

$$\text{Gross income multiplier} = \frac{\text{Selling price}}{\text{Gross income}}$$

Value = Gross income multiplier × Gross income

- DCF method
  - If NOI is expected to grow at a constant rate

$$\text{Value} = \frac{\text{NOI}_1}{(r-g)}$$

  - If property is expected to generate income for a specific holding period before being sold at the end of the holding period, value property as the sum of the PV of income stream and sale price (use direct cap method to estimate sale price or terminal value)

$$\text{Terminal value} = \frac{\text{NOI for the first year of ownership for the next investor}}{\text{Terminal cap rate}}$$

- Cost approach
  - Appraised value = Land value + Building value.
  - Building value = Replacement cost + Developer's profit – Total depreciation.
- Sales comparison approach: calculate average adjusted price per square foot from comparable properties and use this to value property
- Real estate indices
  - Appraisal-based indices: appraisal values lag transaction prices when market shifts suddenly.
  - Transaction-based indices: repeat sales and hedonic indices.
- Loan to value ratio

$$\text{LTV ratio} = \frac{\text{Loan amount}}{\text{Appraised value}}$$

- Debt service coverage ratio

$$\text{DSCR} = \frac{\text{NOI}}{\text{Debt service}}$$

- Equity dividend ratio (cash-on-cash return)

$$\text{Equity dividend rate} = \frac{\text{First year cash flow}}{\text{Equity investment}}$$

REITS

- Net asset value (NAV) approach
  - Estimate value of operating real estate by capitalizing NOI (exclude non-cash rents).
  - Total NAV = Value of operating real estate + Value of other tangible assets – Value of liabilities.
  - NAV per share = Total NAV ÷ Number of shares outstanding.
- Price to funds from operations ratio

Accounting net earnings
Add: Depreciation charges on real estate
Add: Deferred tax charges
Add (Less): Losses (gains) from sales of property and debt restructuring
Funds from operations

- Price to adjusted funds from operations ratio

Funds from operations
Less: Non-cash rent
Less: Maintenance-type capital expenditures and leasing costs
Adjusted funds from operations

- EV to EBITDA ratio: EBITDA can be computed as NOI minus G&A expenses
- DCF valuation approach: use dividend discount model as REITs pay dividends

PRIVATE EQUITY

- Sources of value creation: reorganizing investee company, raising higher levels of debt, aligning interests of management with PE firm
- LBO transactions
  - Significant debt used to finance purchase.
  - Exit value = Initial cost + Value creation from earnings growth + Value creation from multiple expansion + Value creation from debt reduction.
- Venture capital transactions
  - Pre-money valuation (PRE) = agreed value of company prior to a round of financing.
  - Post-money valuation (POST) = value of company after the round of financing (I).
  - POST = PRE + I.
  - Proportionate ownership of VC investor = I ÷ POST.
- Exit routes: IPO (highest valuation), secondary market sale, management buyout, liquidation (lowest valuation)
- Private equity fund performance
  - Gross IRR: based on cash flows between fund and portfolio companies.
  - Net IRR: based on cash flows between fund and limited partners (return to investors).
  - PIC (Paid-in capital): ratio of invested capital to committed capital.
  - DPI (Distributed to paid-in): ratio of cumulative distributions paid to LPs to cumulative invested capital.
  - RVPI (Residual value to paid-in): ratio of LPs' holdings held with the fund to cumulative invested capital.
  - TVPI (Total value to paid-in): sum of DPI and RVPI.
- Basic venture capital method (in terms of NPV)
  - Step 1: Post-money value (POST)

$$\text{Post-money value} = \frac{\text{Exit value}}{(1+\text{Required rate of return})^{\text{Number of years to exit}}}$$

  - Step 2: Pre-money value (PRE): PRE = POST – Investment.
  - Step 3: Ownership proportion of VC investor = Investment ÷ POST.
  - Step 4: Shares to be issued to VC investor

$$\text{Shares to be issued} = \frac{\text{Proportion of venture capitalist investment} \times \text{Shares held by company founders}}{\text{Proportion of investment of company founders}}$$

  - Step 5: Price per share

$$\text{Price per share} = \frac{\text{Amount of venture capital investment}}{\text{Number of shares issued to venture capital investment}}$$

COMMODITIES

- Spot and futures pricing
  - Contango: futures price > spot price.
  - Backwardation: spot price > futures price.
- Insurance theory (theory of normal backwardation): futures market will be in backwardation normally because producers sell futures to lock in prices so that revenues are more predictable
- Hedging pressure hypothesis: if consumers (producers) have greater demand for hedging, the futures market will be in contango (backwardation)
- Theory of storage
  - Futures price = Spot price + Storage costs – Convenience yield.
  - Convenience yield is inversely related to inventory size and general availability of commodity.
- Components of futures returns: price return, roll return, collateral return
- Commodity swaps: excess return swap, total return swap, basis swap, variance swap, volatility swap

PORTFOLIO MANAGEMENT

EXCHANGE-TRADED FUNDS

- ETFs are created in primary OTC market between ETF issuer (who provides ETF shares) and authorized participants (APs - who provide basket of stocks). ETFs trade intraday on secondary stock exchanges.
- APs act as market makers and can purchase new ETF shares by contributing further to a creation basket or can redeem ETF shares via a redemption basket.
- Sources of tracking error include fees and expenses, sampling, variant holdings (e.g. depository receipts), index changes, fund accounting practices, regulatory and tax requirements, and asset manager operations.

MULTIFACTOR MODELS

- Arbitrage pricing theory

$$E(R_P) = R_F + \lambda_1\beta_{P,1} + \ldots + \lambda_K\beta_{P,K}$$

- Carhart four-factor model

$$E(R_p) = R_F + \beta_{p,1}\text{RMRF} + \beta_{p,2}\text{SMB} + \beta_{p,3}\text{HML} + \beta_{p,4}\text{WML}$$

- Active return

$$\text{Active return} = R_p - R_B$$

Active return = Return from factor tilts + Return from security selection

- Active risk is the standard deviation of the active return

$$\text{Active risk squared} = s^2(R_p - R_B)$$

Active risk squared = Active factor risk + Active specific risk

MARKET RISK

- VaR: minimum loss over a particular time period with a specified probability
- Parametric method
  - VaR estimate based on return and standard deviation, typically from normal distribution

$$E(R_P) = \sum_{i=1}^{n} w_i R_i$$

$$\sigma_P = \sqrt{w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_i\sigma_i w_j\sigma_j\rho_{i,j}}$$

$$\text{Unannualized } \sigma_P = \text{Annual } \sigma_P / \text{No. of days}^{0.5}$$

- Historical simulation: returns are ranked lowest to highest, VaR is determined for required confidence interval
- Monte Carlo simulation: employs user -developed assumptions to generate a distribution of random outcomes
- Conditional VaR: average loss expected outside confidence limits
- Incremental VaR: change in VaR if a position within the portfolio changes
- Marginal VaR: change in VaR for a marginal change in portfolio positions
- First- and second-order yield effects on bond price

$$\frac{\Delta B}{B} = -D\frac{\Delta y}{1+y} + \frac{1}{2}C\frac{(\Delta y)^2}{(1+y)^2}$$

- Impact of delta and gamma on call option price

$$c + \Delta c \approx c + \Delta_c\Delta S + \frac{1}{2}\Gamma_c(\Delta S)^2$$

- Sensitivity risk measures can complement VaR because (1) they address shortcomings of position size measures, and (2) they do not rely on history
- Scenario risk measures can complement VaR because (1) they can overcome any assumption of normal

distributions, and (2) a portfolio's most concentrated positions can be stress tested

ECONOMICS AND INVESTMENT MARKETS

- Inter-temporal rate of substitution (ITRS)
  - Ratio of the marginal utility of consumption in the future to the marginal utility of consumption today.
  - ITRS is inversely related to real GDP growth.
  - ITRS is inversely related to the one-period real risk-free rate.
  - Covariance between ITRS and expected future price of a risky asset is negative, resulting in a positive risk premium.
  - The larger the negative covariance, the higher the risk premium.
- Real default-free interest rates are:
  - Positively related to GDP growth rate.
  - Positively related to expected volatility of GDP growth.
- Taylor rule for short-term interest rates

$$pr_t = \iota_t + \pi_t + 0.5(\pi_t - \pi_t^*) + 0.5(Y_t - Y_t^*)$$

Where
pr_t = policy rate at time t
ι_t = real short-term interest rates that balance saving and borrowing
π_t = inflation
π_t^* = the inflation target
Y_t and Y_t^* = logarithmic levels of actual and potential real GDP, respectively

- Break-even inflation rate: difference between yield on a zero-coupon default-free nominal bond and the yield on a zero-coupon default-free real bond (includes expected inflation and risk premium for uncertainty over future inflation)

ACTIVE PORTFOLIO MANAGEMENT

- Sharpe ratio

$$SR_P = \frac{R_P - R_f}{STD(R_P)}$$

- Information ratio

$$\text{Information ratio (IR)} = \frac{\text{Active return}}{\text{Active risk}} = \frac{R_A}{\sigma(R_A)} = \frac{R_P - R_B}{\sigma(R_P - R_B)}$$

- Optimal portfolio construction
  - Sharpe ratio of combination

$$SR_P^2 = SR_B^2 + IR^2$$

  - Optimal level of active risk for unconstrained portfolios

$$\sigma^*(R_A) = \frac{IR}{SR_B}\sigma(R_B)$$

- Full fundamental law

$$E(R_A) = TC\ IC\ \sqrt{BR}\ \sigma_A$$

$$\frac{E(R_A)}{\sigma(R_A)} = \text{Information ratio (IR)} = TC * IC * \sqrt{BR}$$

- Independence of investment decisions
  - *BR* does not equal *N* when (1) active returns between individual assets are correlated, or (2) forecasts are not independent from period to period

$$BR = \frac{N}{1 + (N-1)\rho}$$

ALGORITHMIC TRADING

- Execution algorithms: break down large trades into smaller sizes to minimize trading impact, e.g. VWAP, market participation, implementation shortfall
- High-frequency trading algorithms: find and execute opportunistic, profitable trades, e.g. event-driven algorithms, statistical arbitrage algorithms
- Market fragmentation (same instrument traded in multiple markets): liquidity aggregation creates a "super book" of quote and depth across many markets while smart order routing introduces orders in markets offering best prices and favorable market impact